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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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# VELOCITY DISTRIBUTION FOR LOCAL SUPERSONIC REGIONS ON THIN PROFILES

by K. Oswatitsch

(Translated by M. D. Friedman from ZAMM, Jan. – Feb. 1950)

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# VELOCITY DISTRIBUTION FOR LOCAL SUPERSONIC REGIONS ON THIN PROFILES

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## 1. Basic Equations

It is known that the stationary, irrotational, two dimensional flow around flat bodies which is treated in the following can be described for small Mach Numbers M (ratio of stream speed w to sound speed c) by the Laplace equation and for supersonic velocity by the wave equation. The mathematical problem accordingly changes, for an increase of the Mach number in the free-stream region of $M_\infty$, from an elliptic to a hyperbolic problem. The mixed elliptic-hyperbolic flow problem, which occurs in the neighborhood of sound, has defied all attempts at solution until now. Only exact and approximate streams, which do not yield any parallel streams at infinity, can be sketched.

In the following, a solution of the problem shall again be given which must satisfy the pure mathematician less than the applied mathematician and the aerodynamicist. The detailed work and the comparison with experiment follows in the Physica Acta Austriaca.

With u, v as velocity components and $\bar{x}$, $\bar{y}$ as Cartesian coordinates, the gas dynamic equation (1) which governs the problem andthe equation of irrotationality (2) read

$$(c^2 - u^2)\frac{\partial u}{\partial \bar{x}} - uv\frac{\partial u}{\partial \bar{y}} - uv\frac{\partial v}{\partial \bar{x}} + (c^2 - v^2)\frac{\partial v}{\partial \bar{y}} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \tag{2}$$

For thin profiles, neglecting quadratic terms

$$\frac{v}{u} \ll 1 \quad ; \quad u = w \tag{3}$$

whereby the gas dynamic equation takes the form (4)

$$(1-M^2)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - 2M\frac{v}{c}\frac{\partial v}{\partial x} = 0 \tag{4}$$

For the preceding problem, it is well known that the sound speed and Mach number can be formed as a function of the velocity alone. Here it is sufficient to linearize, whereby it shall be required that the Mach number should be reproduced correctly in the free stream ($M = M_\infty$) and at sound speed ($M = 1$). Therefore

$$1 - M^2 = 1 - M_\infty^2 - \frac{1 - M_\infty^2}{\frac{c^*}{u_\infty} - 1}\left(\frac{u}{u_\infty} - 1\right) \tag{5}$$

with $c$ as critical speed of sound (for $M = 1$). The first term of this development according to $u/u - 1$ is therefore the square of the well-known Prandtl factor.

$$\beta = \sqrt{1 - M_\infty^2} \tag{6}$$

The equations become particularly simple with the following abbreviations:

$$\left.\begin{aligned} & x = \bar{x} \quad ; \quad y = \bar{y}\,\beta \\ & U = \frac{\frac{u}{u_\infty} - 1}{\frac{c^*}{u_\infty} - 1} = \frac{u - u_\infty}{c^* - u_\infty} \quad ; \quad V = \frac{\frac{v}{u_\infty}}{\beta\left(\frac{c^*}{u_\infty} - 1\right)} = \frac{v}{\beta\,(c^* - u_\infty)} \end{aligned}\right\} \tag{7}$$

Observe that $U$ and $V$ in the neighborhood of sound become quantities of $O(1)$. (For $M = 1$, $U = 1$). Only far from the speed of sound are $U$ and $V$ small

With the symbols of (7), equations (4) and (2) now become

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = U\,\frac{\partial v}{\partial x} + 2\left(\frac{u_\infty}{c^*} - 1\right) V\,\frac{\partial V}{\partial x} \tag{8}$$

$$\frac{\partial U}{\partial y} - \frac{\partial V}{\partial x} = 0 \tag{9}$$

The first term of the right side of equation (8) corresponds to the variation of the $(1 - \underline{M}^2)$ factor, the second term to the sum of the two middle terms of the gas dynamic equation (1). These become arbitrarily small when the free stream approaches sound speed ($\underline{u}_\infty \rightarrow c$). The right side of equation (8) vanishes far from the speed of sound. The result corresponds closely then to the Prandtl rule.

Let the profile be given by prescribing $\underline{v}$ on the $\underline{x}$ axis $[(\underline{y} = 0;$ $\underline{v} = \underline{v}_o(x)]$. Then the boundary conditions at infinity and at $\underline{y} = 0$ are

$$\left.\begin{aligned} &\sqrt{x^2 + y^2} \rightarrow \infty : U \rightarrow 0 \quad ; \quad V \rightarrow 0 \\ &y = 0 : \quad V = V(x) = \frac{\dfrac{v_o(x)}{u_\infty}}{\beta\left(\dfrac{c^*}{u_\infty} - 1\right)} \end{aligned}\right\} \tag{10}$$

For asymmetric profiles or the placing of the profile let $\underline{V}_o(\underline{x})$ be prescribed on both sides of the $\underline{x}$-axis.

The expression $\underline{c}^*/\underline{u}_\infty - 1$ is proportional to $\beta^2$ $\left[\beta^2 = (x + 1)\left(\frac{\underline{c}^*}{\underline{u}_\infty} - 1\right) + \ldots\right]$. The boundary conditions (10) therefore correspond not to the prescribing of a profile but to that one group of affine distorted profiles whereby the maximum $\underline{v}_o(\underline{x})$ values, hence the maximum thickness ratio for equal $\underline{V}_o(\underline{x})$ for the greatest $\beta$ values, therefore correspond to smaller free stream Mach numbers. In this work, only such thin profiles shall be treated for which $u_\infty$ lies sufficiently close to $c^*$ so that the term in equation (8) containing the factor $(u_\infty/c^* - 1)$ can be neglected. The system of equations then is

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = U \frac{\partial U}{\partial x} \tag{11}$$

$$\frac{\partial U}{\partial y} - \frac{\partial V}{\partial x} = 0 \tag{12}$$

with the boundary conditions (10).

This limitation is not essential to the accomplishment of the following calculation, yet in this way shall the essential part of the problem be selected and separated from the thickness effect. Profiles which satisfy equations (11) and (12) and the boundary conditions (10) for the same $V_o(x)$ therefore obey a similarity theorem according to which the thicker profile corresponds to the lower free-stream Mach number and the stronger variations of the velocities to the profile. This theorem was exposed almost simultaneously by Guderley, Von Kármán, and K. Oswatitsch without until now being published in detail.

If the stream density $\rho u$ is developed so that, in the free stream $(\rho u = \rho_\infty u_\infty)$, it has the correct value and, corresponding to the Prandtl rule, also the correct tangent at the exact stream-density curve and in addition produces a maximum for $M = 1$ , then the series is

$$\frac{\rho u}{\rho_\infty u_\infty} - 1 = \beta^2 \frac{u}{u_\infty} - 1 \quad - \frac{1}{2} \frac{\beta^2}{\left( \frac{c^*}{u_\infty} - 1 \right)} \left( \frac{u}{u_\infty} - 1 \right)^2 + \dots$$

Hence, with equation (7), it can be written

$$\frac{\frac{\rho u}{\rho_\infty u_\infty} - 1}{\beta^2\left(\frac{c^*}{u_\infty} - 1\right)} = U - \frac{U^2}{2} \tag{13}$$

Thereupon the expression $\underline{U} - \underline{U}^2/2$ represents essentially the stream density, from which it also appears that the gas dynamic equation representing the continuity condition can also be written.

$$\frac{\partial}{\partial x}\left(U - \frac{U^2}{2}\right) + \frac{\partial V}{\partial y} = 0 \tag{14}$$

A perpendicular condensation shock is a jump of supersonic velocity ($\underline{U} > 1$) on a value $\hat{\underline{U}} < 1$ for the same stream density. This follows from

$$U - \frac{U^2}{2} = \hat{U} - \frac{\hat{U}^2}{2} \tag{15}$$

$$U - 1 = 1 - \hat{U} \tag{16}$$

This is the well-known equation for a near-sound perpendicular shock -- only such come within the confines of this theory of thin profiles -- which says that the arithmetic mean of the velocity before and behind the shock is equal to the sound speed.

One is easily convinced that in the system of equations (11), (12) the characteristic properties of a mixed elliptic-hyperbolic problem are completely obtained. The system moreover arises from no uncontrollable omissions. Finally, it gives relations for the condensation shock equations (15) and (16) and the exact description of the procedure for the thinnest in the group of -- corresponding to the similarity law -- affine distorted profile.

## 2. An Integral Equation

Now a velocity potential $\underline{U} = \underline{\Phi x}$, $\underline{V} = \underline{\Phi y}$ can be introduced into the system (11), (12) in the usual way. In the equation

$$\Phi_{xx} + \Phi_{yy} = U \frac{\partial U}{\partial x} \tag{17}$$

the right side represents the source distribution in a Poisson equation. The Prandtl rule, to which would correspond the Laplace equation ($U\, \partial U/\partial x = 0$) and a linear approximation of the stream density, presumes large stream densities $\rho u$. The effective stream with its smaller stream density corresponds to a Prandtl stream with a source distribution in the whole field. Thereby the increase of the velocity corresponds to an increase of the displacement and consequently a source. The source effect vanishes as the square of the distance from the body and the attenuation far away obeys the same laws as that for the Prandtl rule or also for the incompressible stream.

The solution of the Poisson equation for given source distribution is known (Courant-Hilbert, Vol. II, page 230). To the source in equation (17) is added one more source condition on the $x$ axis which gives the desired $v$-distribution. For a symmetrical profile, not at incidence, in order that the boundary conditions at $y = 0$ be the same as for the Prandtl rule or for incompressible flow if the v-distribution of the profile does not change, the source distribution of the field ($U\, \partial U/\partial x$) is likewise symmetrical to the x-axis. In this way for the $U = \Phi_x$ of a symmetric nonpositioned profile (only this case shall be treated here) one obtains

$$U(x,y) = U_p(x,y) - \frac{1}{2\pi} \int_{-\infty}^{+\infty}\!\!\int U(\xi,\eta)\, \frac{\partial U}{\partial \xi}\, \frac{\xi - x}{(\xi - x)^2 + (\eta - y)^2}\, d\xi\, d\eta \tag{18}$$

Here $U_p(x,y)$ is the velocity distribution attained by means of the Prandtl rule

$$U_p(x,y) = -\frac{1}{\pi} \int_{-\infty}^{\infty} V_o(\xi)\, \frac{\xi - x}{(\xi - x)^2 - y^2}\, d\xi \tag{19}$$

In equation (19) the Cauchy principal value is to be taken on $\underline{y} = 0$. The same holds for the double integral of equation (18) on the line $\underline{\eta} = \underline{y}$. It is defined by means of

$$\left.\begin{aligned} &\frac{1}{2\pi}\int_{-\infty}^{\infty}\int U\,\frac{\partial U}{\partial \xi}\,\frac{\xi - x}{(\xi-x)^2 + (\eta-y)^2}\,d\xi\, d\eta = \\ &\frac{1}{2\pi}\lim_{\xi \to 0}\left\{\int_{\xi=-\infty}^{x-\xi}\int_{\eta=-\infty}^{\infty} U\,\frac{\partial U}{\partial \xi}\,\frac{\xi - x}{(\xi-x)^2 + (\eta-y)^2}\,d\eta\, d\xi \; + \right. \\ &\left. \int_{x+\xi}^{\infty}\int_{-\infty}^{\infty} U\,\frac{\partial U}{\partial \xi}\,\frac{\xi - x}{(\xi-x)^2 + (\eta-y)^2}\,d\eta\, d\xi \right\} \end{aligned}\right\} \qquad (20)$$

which is to be particularly noted for the following calculations.

In the vicinity of the maximum thickness of the body the double integral is found to be negative in order that the desired velocity distribution $\underline{U}(\underline{x},\underline{y})$ will be represented there as the Prandtl distribution $\underline{U}_p$ increased by an addition in which, to be sure, the desired distribution is again contained. The problem is reduced to the solution of a singular integral equation. Now, if the double integral is integrated by parts, the following integral equation, taking into account equation (20), is gained

$$U(x,y) = U_p(x,y) + \frac{U^2}{2}(x,y) - \frac{1}{2\pi}\int_{-\infty}^{\infty}\int \frac{U^2(\xi,\eta)}{2}\,\frac{(\xi-x)^2 + (\eta-y)^2}{[(\xi-x)^2 + (\eta-y)^2]^2}\,d\xi d\eta \qquad (21)$$

again with the principal value, defined according to equation (20), for the double integral.

The integral equation (21) represents the fundamental equation of this work. The double integral has here no jump at the jump points of $\underline{U}$ and is, moreover, positive in the vicinity of the maximum thickness of the body, but smaller than $\underline{U}^2(x,y)/2$. The desired velocity distribution can be represented according to equation (21) as $U_p$ increased by an expression that consists of $U^2/2$ and the double integral. Likewise, however, the stream density $\underline{U} - \underline{U}^2/2$ reduced by $U_p$ can be calculated around the double integral. The first representation has the advantage that the velocity is yielded from a relatively small addition to $U_p$, while the difference of $\underline{U}_p$ and $\underline{U} - \underline{U}^2/2$ at the important point proves about twice as great. This difference must, near-sound, also be calculated very accurately since otherwise the velocity can only be calculated very inaccurately from the stream density or yields unusable values of the stream density ($\underline{U} - \underline{U}^2/2$ doesn't exceed the value 0.5). Besides the calculation of $\underline{U}$ from $\underline{U} - \underline{U}^2/2$ is double-valued. However, the advantage also lies in calculating the stream density before calculating the velocity out of equation (21) -- In this way, the jump in the shock is reproduced in the correct quantity because the double integral shows there a continuous course.

Interesting, above all, is the velocity distribution on the profile ($\underline{y} = 0$) for which with $\underline{U}(0,\underline{x}) = \underline{U}_o$ holds

$$U(0,x) = U_o = U_{po} + \frac{U_o^2}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty}\int \frac{U^2(\xi,\eta)}{2} \frac{(\xi-x)^2 - \eta^2}{[(\xi-x)^2 + \eta^2]^2}\, d\xi d\eta \qquad (22)$$

Still the knowledge of the velocity distribution in the whole field is also necessary, to calculate $U_o$.

It is no error when $\underline{U}$ with equations (21) and (22) is represented also in the local supersonic region in a way corresponding to an elliptic type differential equation. However an iteration, for the local supersonic region prohibits in this way that starting from the Prandtl solution $\underline{U}_p$ the values be introduced into the right side of equation (21) in order to obtain the next approximation for $\underline{U}$. This procedure corresponds to the Rayleigh-Jantzen method which, as is well known, does not lead to the desired supersonic fields.

An exact solution of equation (21) which corresponds to the exact calculation of the streams on a thin profile must be very difficult. An iteration of equation (21) starting from a good approximation of the result would be, on the contrary, conceivable.

### 3. Approximate Solution of the Integral Equation

The velocity distribution $\underline{U}$ on a line $\underline{x}$ = constant has always a characteristic course. It falls from the value $\underline{U} = \underline{U}_o$ with an initial tangent $[(\partial \underline{U}/\partial \underline{y})_o = d\underline{V}_o/d\underline{x}]$ which is given by the wall curvature, becomes flatter for larger y-values and vanishes far outside as $1/\underline{y}^2$. While in the vicinity of the wall, the wall curvature is decisive there is yielded a condition, for the course farther outside, from the integration of the continuity conditions with respect to $\underline{y}$. Here (for $\underline{y} \geq 0$) the equation

$$U(x,y) = \frac{U_o(x)}{\left(1 + \frac{y}{b}\right)^2} \tag{23}$$

should be used in that $\underline{U}_o$, the desired velocity on $\underline{y} = 0$ and $\underline{b}$, is a function $\underline{b}(\underline{x})$ which can be so chosen that the irrotationality on $\underline{y} = 0$ or the continuity condition in integral form or both conditions will only be partly fulfilled. By increasing the expenditure of work the approximation of the velocity distribution can naturally be improved in addition by the assumption of a second parameter.

Equation (23) substituted in the double integral of equation (22) permits the integration over $\underline{\eta}$ and gives an approximate integral equation for $\underline{U}_o(\underline{x})$

$$U_o = U_{po} + \frac{U_o^2}{2} - \int_{-\infty}^{\infty} \frac{U_o^2(\xi)}{2b(\xi)} E\left(\frac{\xi - x}{b - (\xi)}\right) d\xi \tag{24}$$

with the rather complicated functional form for $E(z>0)$

$$E(z) = \frac{4}{\pi(Hz^2)^5} \left\{ \frac{\pi}{2} z(5 - 10z^2 + z^4) - (1 - 10z^2 + 5z^4) \ln z - \frac{1}{12}(Hz^2)(25 - 71z^2 - z^4 - z^6) \right\} \tag{25}$$

For $z<0$ a separate determination of $E(z)$ is necessary. The function $E$ is symmetrical in $z$ (fig. 1), increases logarithmically, without limit for $z = 0$ and vanishes entirely at infinity as $1/z^2$ just as the perturbation velocity caused by a body in a parallel stream.

For the point $x = \xi$ , the Cauchy principal value should again be taken. The singularity of E at this point is logarithmic, therefore causing no difficulties.

The problem is thereby reduced to the solving of an approximate integral equation for $U_o(x)$ whereby the boundary conditions shrink, by means of the Prandtl-distribution to $U_{po}(x)$ , in the calculation. With respect to the iteration possibility the same holds for equation (24) as for equation (21).

To calculate the velocity distribution in the region of the high velocities ($U>0$), because of the strong attenuation of $E$ , the low velocity regions ($U<0$) in the vicinity of the fore and aft stagnation points play a subordinate role. It will therefore be important for the calculation of the integral in equation (24) above all, to copy correctly the velocity distribution in the neighborhood of the maximum velocity. Also, above all, since $U_o$ enters as a square in the integrand. If no condensation shocks occur then $U$ has in the vicinity of the maximum a parabolic course (fig. 4a). For condensation shocks with large jump (fig. 4e) an approximation of the $\underline{U}$-distribution by half a parabola is sufficient since the lower velocities behind the shock in the integral of equation (24) are not important. Only for weak shocks is an approximation by means of half a parabola on both sides of the shock necessary. For the value of the integral in equation (24), the function $\underline{b}(\underline{x})$ is further of value, as representing a measure for the height of the disturbance region. Now the attenuation of the disturbance is again dependent on the average profile properties. Therefore it is necessary to calculate with an average constant $\bar{b}$. The correctness of this will be only later confirmed.

A parabolic peak or half a parabolic peak then has three essential parameters. The greatest height given by $\underline{U}_{om}$, the width $\underline{x}_o$ and its position given perhaps by the coordinate $\underline{x}_m$, its maximum. With this, it is given by

$$U_o = U_{mo}\left[1 - \left(\frac{x - x_m}{x_o}\right)^2\right] \tag{26}$$

which statement shall hold only for $U_o \geq 0$, therefore $|x - x_m| \leq x_o$.

After the substitution of equation (26) in equation (24) and the accomplishment of the integration the integral equation can be satisfied, for given $\bar{b}$, at exactly three points corresponding to the three available parameters, out of which $\underline{U}_{mo}$, $\underline{x}_m$ and $\underline{x}_o$ can be determined. The approximation (26) will, in the following, be employed solely as a true velocity distribution but only to calculate the difference between $\underline{U}_o$ and $\underline{U}_{po}$ and also the difference between $\underline{U}_o - \underline{U}_o^2/2$ and $\underline{U}_{po}$. The first method can be applied, according to the required knowledge of $U_o^2/2$, only there where $\underline{u}_o$ is given by equation (26) therefore, above all, in the supersonic region. The second method provides all values, will not be applied usefully however for values near $\underline{\bar{U}} = 1$.

For constant $\bar{b}$, only $\underline{x}/\bar{b}$ and with/also only the value $\underline{x}_o/\bar{b}$ occurs for the computation of the integral in equation (24). There can therewith be substituted:

$$\left.\begin{aligned} \int_{-\infty}^{\infty} \frac{U_o^2}{2\bar{\bar{b}}} E\left(\frac{\xi - x}{\bar{b}}\right) d\xi &= \frac{U_{mo}^2}{2} f_1\left(\frac{x - x_m}{\bar{\bar{b}}}, \frac{x_o}{\bar{\bar{b}}}\right) \\ \frac{U_o^2}{2} - \int_{-\infty}^{\infty} \frac{U_o^2}{2\bar{\bar{b}}} E\left(\frac{\xi - x}{\bar{b}}\right) d\xi &= \frac{U_{mo}^2}{2} f_2\left(\frac{x - x_m}{\bar{\bar{b}}}, \frac{x_o}{\bar{\bar{b}}}\right) \end{aligned}\right\} \qquad (27)$$

The functions $f_1$ and $f_2$ are depicted for the parabolic peak and the half a parabolic peak in figures 2 and 3. They can be used to calculate a wholly arbitrary profile so long as one is satisfied with the approximation (23) in the field for constant average $\bar{b}$ and with equation (26) to calculate the above functions. It is thereby especially astonishing, that for the half-parabolic peak, the value at the maximum hardly depends on the value $x_o/\bar{b}$ therefore also hardly on $\bar{b}$ itself, as a certain carelessness in the choice of parameters justifies.

## 4. Application to the Biconvex Profile

Figures 4a – 4e give an application to a profile which is also symmetrical relative to the maximum thickness and for supersonic speeds also yields therefore a symmetric $\underline{U}$ distribution.[1] Let $\bar{b} = \pi/4$, corresponding to the value of

[1] It is a question of the boundary conditions $\underline{V}_o = 2\tau(1-2\underline{x})$ for $0 \leq \underline{x} \leq 1$; therefore, essentially about a biconvex profile. Thereby, $\tau$ is a "reduced" thickness ratio that, corresponding to the similarity law, is related to the correct thickness ratio $\tau'$ of the profile in the following way:

$$\pi\tau' = \tau\beta\left(\frac{c^*}{U_\infty} - 1\right)$$

$(\partial U/\partial y)_o$ at the maximum for the Prandtl distribution. For the symmetrical solution (parabola peak), coincidence is required of the maximum and of that point in which $\underline{f}_2$ (equation (26)) vanishes, therefore where $\underline{U} = \underline{U}_{po}$. Besides one knows here that the parabola peak lies symmetrically. For the solution with a condensation shock, the zero point of the half-parabola peak will coincide with the zero of the Prandtl distribution, which yields hardly any error. Coincidence will again be required at the maximum and at $\underline{f}_2 = 0$. The calculalations can be accomplished quickly after which the work of calculating $\underline{f}_1$ and $\underline{f}_2$ is once and for all eased. Consequently, the effect of a change in $\bar{b}$ is also easily studied.

There is, for the ratios which are the basis of figure 4d, a symmetric and an asymmetric pressure distribution. Whether the symmetric one is correctly reproduced will require more accurate study because for the calculation of $\underline{U}_o$ over $\underline{U}_o - \underline{U}_o^2/2$ a symmetric solution is perhaps also yielded which, it is true, exhibits an expansion shock before the maximum thickness and a condensation shock symmetric to it behind the maximum thickness which is therefore thermodynamically inadmissible (dotted curve). Practically, this question is, of course, less interesting, since for values much larger than sound speed, the asymmetric solutions are interesting. For yet higher free-stream velocities, the calculation yields in general no more symmetric solution (figure 4e). Interesting and full of practical meaning is the acceleration of the stream behind the shock. For pure subsonics (figures 4a and 4b) nothing new is yielded.

## 5. Retrospect and Prospect

The theory developed here is obtained according to the setting up of an integral equation (21) while qualitatively correct solution laws which contain available parameters are set in the integral equation and the exact fulfillment of the integral equation is required in one of the parameters corresponding to the number of points. The comparison of theory with experiment of course is still lacking. Since, however, the strong change in the type of velocity distribution in a very small region of Mach numbers of the free stream takes place, it is sure that the well-known drag rise will be well reproduced.

Certainly this theory is yet capable of improvement. However, it is interesting to obtain corresponding solutions for the profile at incidence and for a supersonic stream with local supersonic regions.